Math 2D Multivariable MT 2 Sample Problems

Aaron Chen

****This is in no way indicative of the actual exam!!!****

These problems are just to test yourself with some problems that didn't come out of the book. I'm just using miscellaneous exams from other universities. A few of them are strange or difficult. They're intended mainly as thinking exercises - to think in different perspectives.

***Your best study materials are homeworks, quizzes, and Hamid's Practice Problems!**

1. (a) What is $\frac{d}{dt}F(t, 3t^2, 5)$? Here, F is a differentiable function of 3 variables. Express your answer in terms of the partial derivatives of F.
(b) Find the normal line to the surface $(x+y)^3 - (y+z^2)^5 = 9$ at the point (4,-2,1).

2. Find the maximum and minimum values of the function $(x^2 + 2y^2)^{1/2}$ on the domain (it is a disk) $D = \{(x,y) : (x-1)^2 + y^2 \leq 9\}$.

3. Let

$$f(x,y) = \frac{x+y}{\sqrt{x^2+y^2}}, \quad (x,y) \neq (0,0).$$

(a) Write the equation in polar coordinates.
(b) Does the limit as $(x,y) \to (0,0)$ of $f(x,y)$ exist? (Explain why or why not).
(c) Does the graph of f contain any straight lines? (Perhaps with a single point, namely the origin, missing). Explain why or why not.

4. Every equation of the form $ax^2 + by^2 + cz^2 = d$ where a, b, c, d are nonzero real numbers, will describe one of the following: A hyperboloid of one sheet, a hyperboloid of two sheets, a cone, or an ellipsoid, or an "empty" surface. Explain how to tell, given the numbers a, b, c, d what kind of surface it describes. (Mainly, when do we get the different hyperboloids, cones, and ellipsoids).

5. All I'm going to tell you about this differentiable function f on $\mathbb{R}^2$ is that

$$\nabla f(2,3) = < 4, -1 >, \quad f(2,3) = 7.$$

Answer the following questions for which enough information is given, and explain why not enough information is given for the others.
(a) Is $(2,3)$ a local maximum, a local minimum, or neither, for f?
(b) Find the best approximation you can for $f(2.04, 2.99)$. (It may help to do this after part (c)).
(c) There's one point on the graph of f where we have enough information to find the equation of the tangent plane. What are the coordinates of that point? What is the tangent plane's equation?
(d) Is $f(2,4) \geq 7$?
(e) Find $g'(1)$ where $g(t) = f(2t^3, t^3)$.

6. Show that the following limit does not exist:

$$\lim_{(x,y)\to(1,1)} \frac{x+2y-3}{x+y-2}.$$

Hint: It may help to rewrite this problem as a limit to the origin by translating the variables x, y.

7. Suppose for some function $f(x,y)$ we have that

$$f_x = \frac{\partial f}{\partial x} = \frac{1}{x}, \quad f_y = \frac{\partial f}{\partial y} = \frac{1}{y}.$$

Let $x = r\cos\theta,\ y = r\sin\theta$. Calculate $\partial f/\partial r$ and $\partial f/\partial\theta$.

8. (a) Sketch the surface $x^2 + (y-1)^2 = z^2$.
(b) Find the tangent plane at $(4,4,5)$.

9. Find the maximum and minimum values of the function $f(x,y) = (x-1)^2 + (y-1)^2$ on the unit disk $x^2 + y^2 \leq 1$.

10. Let $f(t) =< 3t^2, 2t^3 >$. Find the length of $f(t)$ between $0 \leq t \leq 2$.

11. Consider the function $f(x,y) = x^4 + y^4 + x^2y^2 - xy + 3$. Find the direction of steepest descent from the point $(1,2)$. (Not part of the question, but: What is the rate of descent?)

12. Suppose we are given $z^2 + 4x + 4y^2 - 24y = x^2 + 2z$. Convert this to the standard form of a quadratic surface. What kind of surface is it? Sketch it.

13. Calculate the arclength of the curve

$$r(t) =< 2t^3/3 + 1/3, t + 7, t^2 + 1 >, \quad -1 \leq t \leq 2.$$

14. What I had intended for Morning Quiz 4: Determine if the following limit exists,

$$\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2(y)}{x^2 + 2y^2}.$$

15. Combining my failed Morning Quiz 4 and Afternoon Quiz 4:
Determine the set of points that f is continuous, where

$$f(x,y) = \begin{cases} \frac{x^2\tan^2(y)}{x^3+2y^3} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0). \end{cases}$$

16. See problems from Sections 13.1,2,4 and 14.1 as those were not touched upon too much.